EFFECTIVE THERMAL CONDUCTIVITY OF GRANULAR MATERIALS

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Some fundamental works whose authors propose formulas to calculate the thermal conductivities of granular materials are presented.

Knowledge of thermophysical properties of granular materials is necessary to solve problems in the most diverse branches of science and engineering. The range of application for granular materials is unusually broad. Granular materials are encountered in the food and cryogenics industries, in foundry work, and in structural engineering. They have gained extensive acceptance as effective heat insulators.

Despite the numerous attempts to derive a theoretical formula to calculate the thermal conductivities of granular materials [1, 2], this problem cannot yet be regarded as having been resolved. It is therefore useful to analyze the basic theoretical research in this field and to establish the directions for future work.

I. **Various Models of the Granular System.** The transfer of heat in dry granular materials is achieved through the following simultaneous processes: molecular gas conductivity in the spaces between particles; convection in the gas interlayers; radiation; conduction through the solid particles, as well as conduction from particle to particle, through direct contact between them.

Virtually all the proposed formulas for the calculation of the effective thermal conductivities of granular materials have been derived on the basis of the simulation of real structures. The applicability of a given formula is governed by the correlation existing between the adopted model and the real structure, as well as by consideration of all means of heat transfer.

Below, as part of a historical review, we examine some basic models proposed by various authors. Analysis of these projects demonstrates that in studying

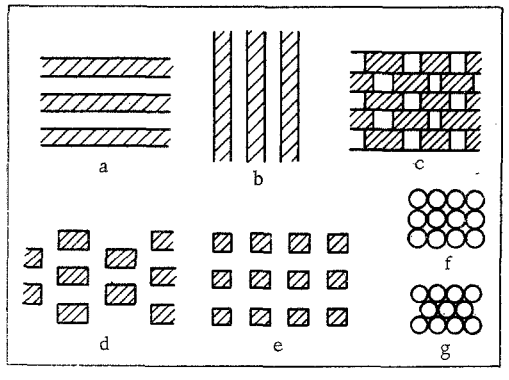

Fig. 1. Various models of granular material. Infinite plates normal (a) and parallel (b) to heat flow; system of bars $p < 50\%$ (c) and $p > 50\%$ (d); system of cubes (e), spheres with cubic (f) and tetrahedral (g).

the processes of heat transfer through granular materials most of the authors—in explicit or implicit form—proceeded in the following natural ways:

a) Some structure of an unbounded granular medium was selected, with the assumption that the structure is ordered or, in other words, the structure of the granular medium exhibits a long-range order [3].

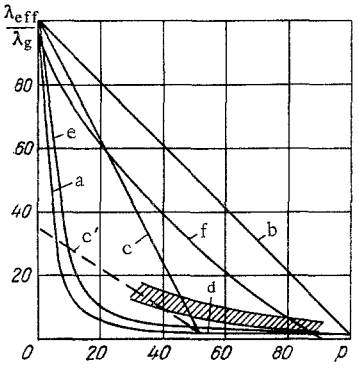

Fig. 2. Comparison of experimental data on effective thermal conductivity of granular materials (hatched region) with calculations according to formulas by Krischer (a, b), Bernstein (c, c', d), Russel (e) and Nekrasov (f). The case $\lambda_s/\lambda_g = 100$ is considered.

b) An elementary cell—that least volume whose repetition in some specific manner for an infinite number of times will yield the original medium with long-range order—is isolated within the medium exhibiting long-range order. It is not difficult to demonstrate that the effective thermal conductivity of the elementary cell is equal to the effective thermal conductivity of the entire medium with long-range order.

With this approach to the analysis of the heat-transfer processes through a granular system, the following assumption is adopted: the effective thermal conductivity of a real (statistical) granular system is equal to the effective thermal conductivity of a system with an ordered structure, if the coefficients of thermal conductivity for the components and of their concentrations are identical for both the real and the ordered systems.

It is obvious that the success of the investigation depends in great measure on the extent to which the selected model of the granular system matches the real structure. The primary attention in the review is therefore centered on the models of a granular system proposed by the various authors. We will consider only

such models which differ significantly from each other. It may be that individual models are excluded from our examination.

Many authors treat granular materials as systems of solid barriers separated by layers of air. The greatest insulating effect is found when an interlayer of air is placed perpendicular to the direction of the heat flow (Fig. 1a). The least insulating effect occurs when the air interlayer is in the direction of the heat flow (Fig. 1b). The formulas for the calculation of the effect of thermal conductivity, corresponding to these two cases, are cited in many sources and are of the following form [4, 5]:

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{100}{\frac{\lambda_g}{\lambda_s}(100-p)+p}; \quad (1a)$$

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{\lambda_s}{\lambda_g}\frac{100-p}{100}+\frac{p}{100}. \quad (1b)$$

For the relationship $\lambda_s/\lambda_g = 100$ (particles of mineral origin in air) these functions are given graphically in Fig. 2 (curves a and b). Here we also see the region of experimental values for the effective coefficient of thermal conductivity in granular materials of mineral origin (cross-hatched) [1, 2, 5–8]. As we can see, the proposed model is somewhat too far removed from the real structure of granular materials.

Bernshtein [9] developed this model somewhat, treating the granular material as a system consisting of unbounded bars placed in a staggered array (Fig. 1c, d). The thermal-conductivity coefficients in this model are calculated on the basis of the following formulas:

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{4p}{100\left(1+\frac{\lambda_g}{\lambda_s}\right)}+\frac{\lambda_s}{\lambda_g}\frac{100-2p}{100},$$

$$p \leqslant 50\%, \quad (2a)$$

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{4(100-p)}{100\left(1+\frac{\lambda_g}{\lambda_s}\right)}+\frac{2p-100}{100}, \quad p \geqslant 50\%. \quad (2b)$$

The functions proposed by Bernshtein are shown graphically by the dashed lines c and d (Fig. 2). The experimental results are found considerably below the top of the curve ($p < 50\%$). The author explains this by the thermal resistance of the thinnest of the air interlayers between the plates at the points at which they are in contact, these interlayers resulting from the roughness of the plates. Calculation with consideration of these clearances yields the c' branch.

A divergence between the experimental and theoretical values that is approximately the same as in the previous cases is given by the Russel model. The author proposes the treatment of the granular material as solid cubes linked to each other by the air interlayers (Fig. 1e), i.e., he assumes an absence of contact and the existence of open pores, even with low porosity values. The Russel formula has the form [10]

$$\frac{\lambda_{eff}}{\lambda_g} = \left\{\left(1-\frac{p}{100}\right)^{2/3}+\frac{\lambda_g}{\lambda_s}\times\left[1-\left(1-\frac{p}{100}\right)^{2/3}\right]\right\}\left\{\left[\left(1-\frac{p}{100}\right)^{2/3}-1+\frac{p}{100}\right]+\frac{\lambda_g}{\lambda_s}\left[2-\left(1-\frac{p}{100}\right)^{2/3}-\frac{p}{100}\right]\right\}^{-1}. \quad (3)$$

Figure 2e shows a graphical representation of this function for $\lambda_s/\lambda_g = 100$.

In most of the remaining works devoted to this question the granular material is regarded as a system of spherical particles of the same size, positioned in a variety of ways. This model apparently best approximates real structures. Certain of the authors proceed from the assumption that the spherical particles are positioned in accordance with the law of cubic symmetry while others assume the particles to be densely packed, i.e., they assume a tetrahedral arrangement. Regardless of the arrangement, porosity is independent of sphere size; it is assumed that the thermal properties of the particles are identical in all directions.

The cubic system is constructed so that the lines connecting the centers of eight adjacent spheres form the faces of the cube. Each particle has six contacts with the other spheres; in the pore between eight of the particles it is possible to inscribe a sphere whose radius is equal to 0.73r, where r is the particle radius. Air occupies 47.67% of the total volume in such a system [3]. Figure 1f shows a single layer of this arrangement. A large quantity of air and a relatively small number of contacts with the surrounding particles make the cubic system unstable. The tetrahedral arrangement is denser, with the lines connecting the centers of three adjacent spheres forming an equilateral triangle (Fig. 1g). The center of the sphere in the next layer is situated above the center of this triangle. Each sphere in the arrangement has twelve contacts with the surrounding spheres; the volume of the spaces makes up 25.95% of the total volume. This is a denser form of the arrangement [3].

The cubic arrangement of the particles in the study of the thermal conductivities of a granular system was adopted by Pokrovskii [11], Schumann and Voss [12], Nekrasov [13], Franchuk [14], et al. The most widely disseminated formula for such a model is the one proposed by Nekrasov:

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{3\pi}{2}\frac{\lambda_s}{\lambda_g}\frac{90-p}{(210+p)^2}100. \quad (4)$$

Formula (4) yields a virtually rectilinear relationship between the coefficient of thermal conductivity and the porosity of the material (Fig. 2f), whereas the experimental function is different in nature. Moreover, in deriving the formulas Nekrasov took into consideration only the resistance of the solid particles and ignored entirely the resistance of air spaces. As a result, the effective coefficient of thermal conductivity for the granular materials is proportional to the thermal-conductivity coefficient for the material of the particles, which is also not confirmed by the experiments. The experiments show that for a given porosity of the granular systems consisting of various mate-

rials, the effective thermal conductivity coefficients do not differ from each other as markedly as the λ_g of the actual materials. This is explained by the relatively small role played by contact-heat transfer in the overall heat-transfer process in granular materials under normal pressure of the filler gas.

The tetrahedral arrangement is more stable than the cubic and, consequently, more probable; here the spherical particles are positioned in the densest possible manner.

Following the tetrahedral scheme, Bogomolov [7] in 1941 proposed the theoretical function $\lambda_{eff} = \lambda(p)$, which until recently was highly regarded, since it provided the best reflection of the experimental variation in $\lambda(p)$. The investigations of Lyalikov [15], Kaufman [16], and the authors of this article [17] were based on the tetrahedral arrangement for the particles of the granular system.

In Bogomolov's model a tetrahedral particle arrangement is assumed; each particle is surrounded by a uniform gas halo whose dimensions increase with increasing porosity; there is no direct contact with a porosity in excess of 26%. In addition, Bogomolov assumes that the transfer of heat in the granular material is achieved primarily by molecular transport from particle to particle through the medium separating them, and he takes into consideration only this form of heat transfer. In addition, he deals only with the transfer of heat through the gas halo, neglecting the numerous pores within the material, and he proposes the following theoretical formula [7]:

$$\frac{\lambda_{eff}}{\lambda_g} = 3\pi \ln \frac{43 + 0.31p}{p - 26}. \qquad (5)$$

Formula (5) which is graphically given by curve I in Fig. 3 provides a qualitatively correct description of the variation in the experimental function $\lambda(p)$ (the cross-hatched area in this figure is the region of experimental values of λ_{eff}/λ_g for particles of mineral origin). This made it possible to assume that such a model reflects rather well the actual structure of the granular systems and makes it possible to establish the relationships characteristic of these. However, we note the following shortcomings of Bogomolov's work:

1. The geometric model of a granular system does not always correspond to a real system (the absence of contacts with a porosity above 26%).

2. Not all of the pores in the granular medium are considered in the determination of heat transfer.

3. The proposed function is applicable to granular materials with low porosity ($p < 60\%$), since a small halo thickness was assumed in the derivation of formula (5).

4. A mathematical error (also noted by Lyalikov [15] exaggerating the final result by a factor of two occurred in the calculation. Correction of this error leads to a drop in the theoretical curve I to position II (Fig. 3).

Apparently those processes not taken into consideration by Bogomolov were fortuitously compensated by the error in the calculations and formula (5) therefore yielded excellent agreement with experiment for a number of materials in the given interval of porosities, thus gaining extensive acceptance in the literature.

The cited shortcomings in the Bogomolov model led to the need for further research into the processes of heat transfer through granular systems. Thus, for example, proceeding from a tetrahedral arrangement of particles in a granular system, Lyalikov introduces contact between the particles. Here it is assumed that the contact is a result of the microirregularities of the particle surfaces [15]. The author assumes that with an increase in porosity the particles will spread apart in the horizontal plane, remaining in contact, while the height of the layer remains constant throughout.

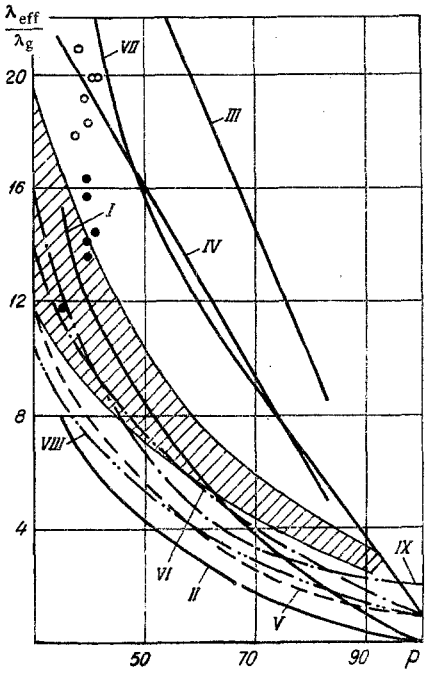

Fig. 3. Graphical representation of various theoretical functions: I and II) Bogomolov; III and IV) Lyalikov; V) Kuni and Smith; VI) Dulnev and Sigalova; VII) Vasil'ev; VIII) Kaganer; IX) Prasolov.

We will demonstrate that these two assumptions are mutually exclusive. If we regard the height of the elementary cell to be constant, on separation of the spheres in the horizontal direction an air space should develop between them. If there is no such clearance and if contact is maintained, some of the spheres must drop to the lower layers, i.e., the height of the layer must diminish. The Lyalikov formula for the calculation of the thermal conductivity in granular materials has the form

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{6(100-p)}{100e}\left(\frac{1}{e}\ln\frac{1+\frac{\delta_0}{r}}{1+\frac{\delta_0}{r}-e} - 1\right),$$

$$e = 1 - \frac{\lambda_g}{\lambda_s}, \qquad (6)$$

where the parameter takes into consideration the transfer of heat through the sphere; δ_0 is the additional gas-layer thickness through whose introduction we give

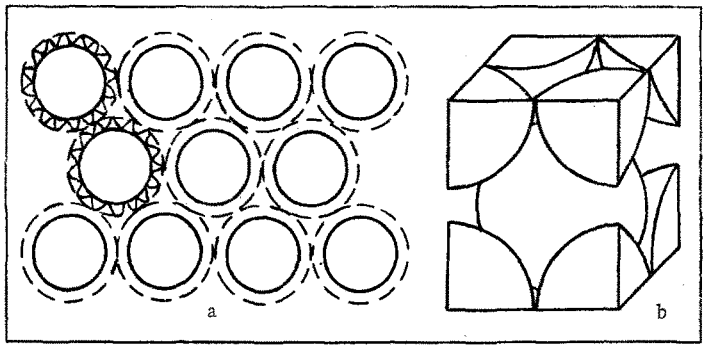

Fig. 4. Representation of granular material as a system of sphere-shaped particles: a) Tetrahedral arrangement of spheres with halo; b) elementary cell.

consideration to the temperature jump occurring at the boundary between the solid and the gas.

Lyalikov, just as Bogomolov, assumed that the transfer of heat took place in the gas between two spheres always in contact. Function (6) is shown graphically by lines III and IV in Fig. 3. As we can see from the formula, λ_{eff}/λ_g is a function not only of the porosity, but also of the particle dimension (r). Curve III has been calculated for particles with d = 2r = 5 mm, while curve IV represents particles with d = 0.5 mm. The same graph shows the experimental values of the effective thermal conductivity from the Lyalikov data (the open circles) [15, 18] and those of other authors (the solid circles) [12, 19–21], for the thermal conductivities of systems consisting of metal particles (d = = 1–5 mm). As we can see, the extension of function (6) to a broad class of granular materials leads to substantial divergence with respect to experimental data.

The authors [22, 23], dealing with a cubic and tetrahedral arrangement of grains, proposed a theoretical formula derived in the assumption that the real granular structures occupy an intermediate position between a loose arrangement and the very densest. The formula for the calculation of the thermal conductivities of unbound granular materials, without consideration of radiation or the transfer of heat through grain contact, has the following form:

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{p}{100} + \frac{\left(1 - \frac{p}{100}\right) 0.9}{\varepsilon + \frac{2}{3}\frac{\lambda_g}{\lambda_s}},$$

$$\varepsilon = \varepsilon_2 + (p - 25.9)(\varepsilon_1 - \varepsilon_2)/21.7. \qquad (7)$$

The magnitudes of the parameters ε_1 and ε_2 were shown graphically as functions of λ_s/λ_g; ε_1 corresponds to a cubic arrangement for uniform spheres (p = 47.6%), while ε_2 corresponds to a tetrahedral arrangement (p = 25.9%). Function (7) for particles of mineral origin (λ_s/λ_g = 100) is shown graphically by curve V in Fig. 3. The numerical coefficients in formula (7) are defined by the geometry of the particles and their mutual location. Function (7) yields values for the effective coefficient of thermal conductivity of the granular materials of mineral origin that are smaller by a factor of 1.5–2 than the experimental data. Better agreement is found for systems of metallic particles. Thus, for a porosity of p = 40% the ratio λ_{eff}/λ_g, calculated according to formula (7) for metallic particles (λ_s/λ_g = = 10^3–10^4), lies within the range 12.5–18.5, which corresponds to the following values for the thermal conductivities of systems of metallic particles in air: λ_{eff} = 0.35 W/m · deg (lead) and λ_{eff} = 0.5 W/m · deg (copper). The experimental thermal-conductivity values for the corresponding materials lie within these limits.

In the model of the granular system proposed by Dul'nev and Sigalova [17] the real particles are replaced by spheres and the region occupied by irregularities is treated as a halo of uniform thickness (α). The sphere-halo system exhibits a tetrahedral arrangement, as shown in Fig. 1g and in Fig. 4a. The elementary cell of such a system may have the form shown in Fig. 4b. Analysis of the process of heat transfer in this cell yields the following relationship between the effective thermal conductivity of granular materials and porosity:

$$26\% \leqslant p \leqslant 75\%,$$

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{1}{A}(X + Z) + \lambda_r + \lambda_c, \quad p \geqslant 75\%,$$

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{1}{A}(X + Z' + V) + \lambda_r + \lambda_c,$$

$$X = 4.45\left(A \ln \frac{A}{A-1} - 1\right),$$

$$Z = 2.23(1.41A - 1), \; Z' = \frac{2.23}{1.41A - 1},$$

$$V = A - \frac{\pi}{2A}, \quad A = \sqrt[3]{\frac{74}{100 - p}}, \qquad (8)$$

where λ_r and λ_c are the radiation and contact components of the thermal-conductivity coefficient.

The term X in these formula characterizes the transfer of heat through the halo, while Z and Z' characterize the heat transfer through the pores and V represents the transfer of heat in additional ways for a porosity of p ≥ 75%. Function (8) without consideration of radiation and transfer of heat through the contacts is shown graphically by curve VI in Fig. 3 (radiation and contact thermal conductivity make up an insignificant fraction (1–5%) of the total thermal conductivity

of mineral-origin granular systems at room temperature and at normal pressure for the filler gas).

Comparison of calculations carried out in accordance with formula (8) and the experimental data showed that this relationship may be extended to the class of polydisperse materials in which the particles themselves are porous [25, 26]. In this case the over-all porosity should be divided into the external porosity p_1 (the pores between the particles) and the internal porosity p_2 (the pores within the particles). The external porosity is set equal to 40–50% (the porosity of systems of large monolithic particles varies within these limits). The porosity of the particles is associated with the over-all and external porosity by the following relationship:

$$p_p = \frac{p - p_1}{100 - p_1} 100.$$

The effective thermal conductivity λ_p of a porous particle is calculated in accordance with the formulas for solid porous materials [24]. Further calculation is performed in accordance with formulas (8) with consideration of the fact that A is a function of the external porosity p_1, while λ_s is replaced by λ_p.

In [27] Vasil'ev extends the model for solid porous systems proposed by Dul'nev [24] to granular materials. The transition to granular materials is accomplished by consideration of the thermal conductivity of the gas clearance at the junction between two particles and the thermal conductivity of the contact. The necessary data on the height of the microirregularities and the contact area are taken by the author [17, 43]. The theoretical formula proposed by Vasil'ev has the following form:

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{\lambda_s}{\lambda_g}\left[\frac{1}{1/(h/L)^2+\Phi} + \frac{\lambda_g}{\lambda_s}\left(1 - \frac{h}{L}\right)^2 + \right.$$

$$\left. + \frac{2}{1 + \frac{h}{l} + \frac{\lambda_s}{\lambda_g}(h/L)^{-1}}\right]. \qquad (9)$$

Here h, l, and L are the characteristic dimensions of the pores and of the elementary cell, while their ratios are functions of porosity [24]. The parameter Φ is a function of the porosity as well as of the relationship between the contact thermal conductivity and the thermal conductivity of the base. Function (9) is shown graphically by curve VII in the summary graph shown in Fig. 3. The significant divergence between the theoretical data and those of the experiment is apparently related to the inadequate agreement between the chosen model and a real granular system.

In all of the above-considered investigations the real granular systems were replaced by idealized structures with long-range order. Studying the processes of heat transfer through chaotic granular systems, Kaganer does not assume long-range order and provides no clear model of the system [28]. In deriving the expression for the effective coefficient of thermal conductivity in such a system the author begins his study with an analysis of the heat transfer Q between two spheres, in analogy with the manner in which this was done in [7, 15, 17].

The effective thermal-conductivity coefficient λ_{eff} of a granular layer is equal to

$$\lambda_{eff} = \frac{Qh}{S\Delta t},$$

where Q is the heat flow through a surface of area S; h is the height of the layer, corresponding to the temperature difference Δt; S is the area of the layer referred to a single grain.

The author assumes that the number N of contacts between the spheres is distributed uniformly in all directions and in one of the directions is equal to N/6. (We note that this assumption is obvious for chaotic systems, whereas for ordered arrangements of grains it may not be satisfied). In this case the relationship between the heat flux Q_1 between two spheres in the presence of a single contact and the flow Q is given by the expression

$$Q = Q_1 N/6.$$

In the following, in determining the area S, M. G. Kaganer makes the implicit assumption of long-range order in the arrangement of the grains. As a matter of fact, the expression

$$p = 1 - \frac{V_3}{hS}$$

contains the assumption that the height h and bed area S of any grain pair in the system, referred to a single grain and to the space around that grain, are identical. In a chaotic system the parameter S for various grain pairs must be different in the general case. Moreover, it is assumed that h = 2r remains constant with a change in the porosity system.

The expression for the effective thermal conductivity of granular systems assumes the form

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{N(100-p)}{200e}\left(\frac{1}{e}\ln\frac{\lambda_s}{\lambda_g} - 1\right), \qquad (10)$$

$$e = 1 - \frac{\lambda_g}{\lambda_s}.$$

We will demonstrate that (10) changes into (6), derived by Lyalikov for a tetrahedral grain arrangement. For this we will assume in formula (6) that $\delta_0 = 0$, since Kaganer failed to take account of the temperature-discontinuity phenomenon, and we will assume the number of contacts to be N = 12, since Lyalikov is dealing with a tetrahedral arrangement. Consequently, until now Kaganer had essentially been examining an ordered system whose model corresponds to the Lyalikov model.

For chaotically positioned particles Kiselev established the relationship

$$N = 11.6\,\frac{100-p}{100}$$

for the case of uniform particle distribution over the volume [29]. This expression for N was subsequently

substituted by Kaganer into (10). This operation makes it possible to regard the system as chaotic.

Formula (10) does not satisfy the passage to the limit $\lambda_{eff} = \lambda_g$ for p = 100%. To eliminate this drawback, Kaganer artificially introduces additional terms into this formula. The final expression for the calculation of the effective thermal conductivity of the granular materials has the form

$$\frac{\lambda_{eff}}{\lambda_g} = \frac{5.8(100-p)^2}{100e}\left(\frac{1}{e}\ln\frac{\lambda_s}{\lambda_g} - 1 - \frac{e}{2}\right) + 1. \quad (10a)$$

The author provides no theoretical foundation for the introduction of these additional terms. Graphically, function (10a) for the case $\lambda_s/\lambda_g = 100$ is given by curve VIII in Fig. 3.

Formula (10a) may also be used to calculate the effective thermal conductivity of polydisperse structures. Here we consider two types of structure:

a) the grains are conglomerates of small spherical particles;

b) the grains are cellular in structure.

In the first case, formula (10a) is used to determine the thermal conductivity λ_g of the grain consisting of small particles, with the same formula used to calculate the thermal conductivity of the entire material (λ_g is substituted for λ_s). In the case of a cellular structure the use of the Rayleigh-Eucken formulas [30] and the Russel formula [10] is suggested for the determination of the thermal conductivity λ_g of the cellular grains, the calculation subsequently being carried out in accordance with formula (10a). Reference [28] provides no specific suggestions as to the determination of the external and internal porosities of the polydisperse system, thus introducing indeterminacy into the calculations.

We note that the analysis of the transfer of heat through the granular system, proposed by Kaganer, is contradictory. As was demonstrated, in determining the parameter N the author regards the system as chaotic, while in determining the function S = S(p) he makes the implicit assumption of a long-range order in the system. Moreover, there is no justification for the changeover from formula (10) to formula (10a).

To calculate the heat transfer in high-porosity (p > > 80%) granular materials, Prasolov treats the granular structure as a system of (solid-gas) parallel planes [33]. Considering the effect of the continuous pores, Prasolov subsequently proposes the function $\lambda_{eff} = \lambda(p)$, applicable to a wide range of porosities [34]:

$$\frac{\lambda_{eff}}{\lambda_g} = \left\{2\left[1 + \frac{(300-2p)^2}{\pi(1.14p-14)100}\left(\frac{\lambda_g}{\lambda_a}\right) + \right.\right.$$

$$+ \frac{(100-p)(14+2p)}{(1.14p-14)100}\left(\frac{\lambda_g}{\lambda_s}\frac{\lambda_g}{\lambda_a}\right) + \left.\left.\frac{14+2p}{\pi\cdot 100}\left(\frac{\lambda_g}{\lambda_a}\right)\right\}\times\right.$$

$$\times\left[\left(\frac{\lambda_g}{\lambda_a}\right) + \frac{\pi(100-p)}{(1.14p-14)}\left(\frac{\lambda_g}{\lambda_s}\frac{\lambda_g}{\lambda_a}\right)\right]^{-1}. \quad (11)$$

The effect of thermal conductivity for granular materials is presented graphically in Fig. 3 as a function of porosity, these functions having been plotted from formulas (5–11), as proposed by various authors. It was assumed in each of the cases that $(\lambda_r + \lambda_c)$ is small in comparison with the molecular thermal conductivity, and that we can assume $(\lambda_r + \lambda_c) = 0$. Moreover, cases were considered in which the particles exhibit a thermal conductivity greater than that of the gas-filler ($\lambda_s/\lambda_g = 100$). It follows from Fig. 3 that function (8) most closely reflects the actual shape of the curve $\lambda_{eff} = \lambda(p)$.

Analysis of the various studies on the effective thermal conductivity of granular systems under conditions of normal gas pressure and low temperatures (not above 20–30° C) permits the following conclusion: the assumption as to the possibility of replacing a real granular system by an ordered structure has been validated experimentally. Also of interest is an analytical study of the process of heat transfer through granular systems with chaotic particle distribution. However, this study should be carried out if it will yield results no more cumbersome and no less exact than the analysis of ordered structures. At the moment, we know of no work in which the system is logically treated as chaotic.

Till now we have been speaking of the effect exerted by the structure on the effective thermal conductivity of a granular system. Here it was assumed that the parameters λ_s, λ_g, λ_r, and λ_c contained in these formulas are known. Let us examine methods for the calculation of these parameters in granular systems.

II. Separate Components of Thermal Conductivity in a Granular System. The thermal conductivity coefficient λ_s for the particles of the material, as well as the thermal-conductivity coefficient λ_0 at normal pressure, are taken from reference literature. According to the molecular-kinetic theory of gases, the thermal conductivity of the gas begins to diminish with a rise in pressure if the mean molecular path Λ is of the same order or higher than the distance δ between the heat-transfer surfaces. We know of the following form for the thermal conductivity of a gas as a function of pressure [31]:

$$\lambda_g = \frac{\lambda_0}{1 + \frac{B}{H\delta}}, \quad (12)$$

where B is the coefficient defining the characteristics of the gas and of the heat-transfer surfaces.

The reduction in the effective thermal conductivity of granular materials with a drop in the pressure of the gas filler is caused by the change in the thermal conductivity of the gas.

Among the earliest works concerned with the investigation of the thermophysical properties of granular materials under conditions of reduced gas-filler pressure we have those of Smolukovski [32]. Examining a powder containing grains of spherical shape whose thermal conductivity is infinitely large in comparison with the thermal conductivity of the gas, he proposed the calculation of the thermal conductivity λ_{eff} of the granular materials as a function of the gas-filler pressure H according to the following formula which should be treated as a generalization of experimental data:

$$\lambda_{eff} = M \lg\left(1 + \frac{r}{\Lambda_0 H_0}\right), \qquad (13)$$

where M is a coefficient which is a function of λ_0 and p. The values of M are presented in [32] for several granular systems.

In an analytical determination of the effective thermal conductivity of a granular system as a function of gas pressure the following method is generally employed: the value of λ_g from (12) [17, 27, 28, 33–39] is substituted into the formula for the effective thermal conductivity of any of the structures. The fundamental difficulties in this case are associated with the determination of the pore dimensions δ of an actual system. Let us examine how this problem is solved by certain authors.

R. S. Prasolov uses the formal apparatus of the molecular-kinetic theory of gases to determine the average pore dimension δ, treating a chaotic structure as a time-fixed pattern of molecular distribution. The expression for δ has the form [33, 34]

$$\delta = d\,\frac{(1.14p - 14)}{6(100 - p)}.$$

Using this expression and (12) it is possible to determine the effective coefficient of thermal conductivity for granular materials in the case of reduced gas-filler pressure from formula (11).

To calculate the thermal conductivities of materials made of powders and loose fibers under conditions of reduced pressures, V. M. Kostylev [40] derived a formula in which the concept of a specific solid-phase material surface is employed. For practical calculations with this formula it is necessary to have available data on the magnitudes of the specific solid-phase surface of powder materials.

Formula (8), proposed by G. N. Dul'nev and Z. V. Sigalova, with consideration of (12)—the thermal conductivity of the gas as a function of pressure—may be employed to calculate the thermal conductivities of granular materials over a wide range of pressure variations in the gas filler. The value of λ_g should be expanded for this purpose in expression (8) as per formula (12). Here it should be borne in mind that for the adopted idealized model of a granular material there exists several types of gas intervals exhibiting various dimensions. In the light of the foregoing, formulas (8) assume the following form [26]:

$$\left.\begin{aligned} &26\% \leqslant p \leqslant 75\% \\ &\lambda_{eff} = \frac{\lambda_0}{A}\left(\frac{X}{1 + \frac{B}{H\delta_{ha}}} + \frac{Z}{1 + \frac{B}{H\delta_p}}\right) + \lambda_r + \lambda_c \\ &p \geqslant 75\% \\ &\lambda_{eff} = \frac{\lambda_0}{A}\left(\frac{X}{1 + \frac{B}{H\delta_{ha}}} + \frac{Z'}{1 + \frac{B}{H\delta_p}} + \right. \\ &\left. + \frac{V}{1 + \frac{B}{H\delta_{cell}}}\right) + \lambda_r + \lambda_c \end{aligned}\right\} \qquad (14)$$

Here $\delta_{ha} = 2r(A - [2/3])$ is the mean halo dimension; $\delta_p = 2r(1.41A - 1)$ is the mean pore dimension; $\delta_{cell} = 2(2)^{1/2}rA$ is the height of the elementary cell.

Formula (14) can be substantially simplified if in the place of δ_{ha}, δ_p, and δ_{cell} we introduce the average value of the pore size $\bar{\delta}$, calculated according to the formula

$$\bar{\delta} = \sum_{i=1}^{n} \delta_i k_i \Big/ \sum_{i=1}^{n} k_i, \quad i = \text{ha, p, and cell,}$$

where k_i is the coefficient by means of which we account for the relative effect of the i-th gas interval on the coefficient of effective thermal conductivity, i.e., heat conduction resulting exclusively from molecular transport.

It is obvious that with this definition of k_i

$$\sum_{i=1}^{n} k_i = 1.$$

The calculations yielded the following values of the parameter $\bar{\delta}$:

$$26\% < p < 75\% \quad \bar{\delta} = 2.1r\left(A - \frac{2}{3}\right),$$

$$p > 75\% \quad \bar{\delta} = 2.8r\left(A - \frac{2}{3}\right).$$

Formulas (14) then assume the form

$$\left.\begin{aligned} &26\% \leqslant p \leqslant 75\% \\ &\lambda_{eff} = \frac{\lambda_0}{A}\,\frac{X + Z}{1 + \frac{B}{H\bar{\delta}}} + \lambda_r + \lambda_c \\ &p > 75\% \\ &\lambda_{eff} = \frac{\lambda_0}{A}\,\frac{X + Z' + V}{1 + \frac{B}{H\bar{\delta}}} + \lambda_r + \lambda_c \end{aligned}\right\} \qquad (15)$$

Calculations according to (14) and (15) coincide satisfactorily with the experimental data of an extensive class of granular materials [41].

With consideration of (12), M. G. Kaganer uses formula (10a) to calculate the effective thermal conductivities of granular materials for various gas-filler pressures. This theoretical formula assumes the following form:

$$\lambda_{eff} = \frac{\lambda_0}{1 + \frac{Б}{H}}\left[Д + C\ln\left(1 + \frac{Б}{H}\right)\right] + \lambda_a, \qquad (16)$$

where λ_a denotes the contact and radiative components of the thermal-conductivity coefficient (determined experimentally).

Although expressions for the constants Д and Б are presented in [28], the author suggests that their values be determined experimentally. Calculation with formula (16), including the theoretical determination of the constants Д and Б, yields a deviation from the experimental data reaching as high as 30–100% (for example, Kannuluik and Martin [42]).

Let us turn to an examination of the contact component λ_c of the effective thermal conductivity of gran-

ular materials. Contact heat transfer is a function of the physical and mechanical properties of the material, as well as of the contact area which is governed by the pressure force exerted on the material [43]. In the case of a granular material poured out freely, this force will be the weight of the layers on top; on application of an external load, the contact area and consequently, the contact thermal conductivity will increase. The contact thermal conductivity for granular materials as a function of the external load Δ can be approximately calculated from the formulas proposed in [28, 44].

Kaganer calculates the thermal resistance of two grains that are in contact by means of the formulas for the contact between semi-infinite bodies [43] and demonstrates that for a spot-contact radius a considerably smaller than the particle diameter d ($d/a > 10$) such a substitution is permissible. He defines the spot-contact radius according to the Hertz formula [45]. Using his model of a granular material in the form of chaotically distributed spherical particles, Kaganer suggests the following formula for the calculation of the contact thermal conductivity as a function of external load [28]:

$$\lambda_c = \frac{3.37\,(100-p)^{4/3}\,\lambda_s\,\Delta^{1/3}}{100^{4/3}E^{1/3}} + \lambda_{free}, \qquad (17)$$

where λ_{free} is the thermal conductivity of the granular material in the state of free flow.

Semiempirical formulas have been derived in [44] to calculate the contact thermal conductivity of granular materials as functions of the external load:

for loads from 0 to $3 \cdot 10^5$ N/m^2*

$$\lambda_c = \lambda_{free} + \frac{\lambda_s}{1.6\cdot 10^5}\,\frac{1}{A}\,\Delta^{2/3}\,k_m, \qquad (18a)$$

for loads from $3 \cdot 10^5$ to $16 \cdot 10^5$ N/m^2

$$\lambda_c = \lambda_{free} + \frac{\lambda_s}{E^{4/3}}\,\frac{1}{A}\,\Delta^{4/3}\,k_b. \qquad (18b)$$

Here A is a function of the porosity determined according to formula (8), and k_m and k_b are empirical coefficients which are functions of the external load. As demonstrated in [44], for particles of mineral origin ($\lambda_s < 1$–2 W/m · deg) $\lambda_{free} = (2$–$15) \times 10^{-3}$ W/m · deg, and for systems of metallic particles $\lambda_{free} = (3$–$10) \times 10^{-2}$ W/m · deg. The values of the coefficients k_m and k_b are also given in [44].

It follows from formulas (17)–(18) that the contact thermal conductivity of a granular material is independent of the particle dimensions, but is defined exclusively by the mechanical (E) and the physical (λ_s) properties of the particle material, the porosity (p) of the system, and the external load (Δ). Comparison of calculations according to (17) and (18) with numerous experiments [25, 46, 47] demonstrates that these formulas can be used for tentative calculations. The experimental and theoretical data in this case may differ by factors of 2–3.

An expression is given in [48] for the heat flux through thermal insulation as a result of contact conductivity in the state of free-flow coverage. However, the application of the proposed formula is difficult since it contains an additional parameter—the coefficient of the internal friction of the granular material—whose determination represents an independent problem.

The experimentally determined λ_{free} includes the radiation component. As mentioned earlier, at room temperature this component is small for granular materials. However, with a rise in temperature its value increases.

An approximate evaluation of the radiation component of the effective thermal conductivity of granular materials is presented in a number of papers. A review of these is presented in the monograph [2]. In studying radiative heat exchange, as a rule, we treat the granular material as a system of nontransparent screens and the expression for the radiative component λ_r has the following form:

$$\lambda_r = k\varepsilon^3 T^3 \delta,$$

where k is a numerical coefficient which is a function of the adopted model of the granular system. No consideration is given in these works to the scattering and absorption of particle radiation. The indicated model for radiative transport is somewhat too coarse, which serves to explain the unsatisfactory agreement with experiment.

Another approach is possible in the investigation of heat transfer by radiation through a granular system. It may be based on the representation of the granular material in the form of some homogeneous semitransparent medium characterized by integral values for the absorption and scattering coefficients. An analysis of the processes of radiative heat exchange in such media is presented in [49–54]. However, we know of no papers in which this method is employed for an analysis of the radiative component of the thermal-conductivity coefficient for granular materials.

In conclusion we note that the processes of molecular heat transfer through granular materials as standard as well as at reduced pressure have now been studied both theoretically and experimentally in rather complete detail. Contact heat transfer can be evaluated at the moment only tentatively. The processes of radiative heat exchange in granular media, as well as the effect of moisture on the effective thermal-conductivity coefficient, have not been studied so extensively. It is clear that these last two lines in the study of heat transfer require more persistent attention on the part of researchers.

NOTATION

λ_{eff} is the effective thermal conductivity of granular material; λ_g is the thermal conductivity of a gas; λ_0 is the thermal conductivity of a gas at a normal pressure; λ_s is the thermal conductivity of material of particles; λ_c is the contact thermal conductivity; λ_r is the radia-

*Formulas (18) have been taken from reference [44]. In the similar formulas in this paper E and Δ should be used in units of kg/cm^2 rather than in units of N/m^2, as proposed in [44].

tive thermal conductivity; p is the porosity; r is the radius of particles, N is the number of contacts; H is the pressure of gas-filler; δ is the size of pores; Λ_0 is the mean free path of gas molecules at a normal pressure H_0; E is the elasticity modulus of material of particles; Δ is the external load; k_m and k_b are the empirical coefficients; T is the temperature; ε is the emissivity.

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